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GIS-based modelling of topography-induced solar radiation variability in complex terrain for data sparse region

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Solar radiation not only sustains the lives on the Earth, but also creates spatial and temporal variations of hydrological ingredients, such as vegetation, soil moisture, and snow. Precise quantification of spatial solar radiation incident on the Earth's surface which accounts for the topographic modulation, especially in complex terrain, underpins the study of many catchment hydro-meteorological and hydro-ecological processes. Topography is a key parameter that affects the spatial solar radiation pattern across different scales. This article addresses the issue of modelling spatial variability of actual solar radiation caused by topography from the hydrological perspective. Models with different algorithms and different complexities, from the simple empirical equations to process-based physical approach, have been developed to parameterize and calculate the potential radiation (under clear-sky condition) and the actual radiation (under overcast cloudy condition). Based on a review of the general steps of solar radiation modelling and the corresponding models for each step, two models with easily or globally available data for spatial solar radiation modelling in complex terrain, namely, the physically parameterized, remote-sensing-oriented Heliosat-2 model and the sunshine duration-based Angström–Prescott regression model are selected and implemented in a GIS framework. The capability of both models for simulation of cloudy-sky radiation on horizontal surfaces has been verified against observed station data showing an $R^2$ greater than 0.9. The validity of the models for modelling inclined surface is tested by comparing against each other, which has shown a satisfactory agreement and demonstrated that the simple Angström–Prescott method performed reasonably well compared with the more elaborate Heliosat-2 method. Scale sensitivity of the models and the shading effect are examined with different digital elevation model (DEM) resolutions from 30 to 500 m and reveal the existence of a threshold grid size to resolve the topography-induced spatial solar radiation variability. Spatial mapping of potential solar radiation and actual solar radiation has been demonstrated in a small catchment in Southern Germany, with a spatial difference up to 30% in winter and 5% in summer. This may lead to a significant difference for the energy-limited hydrological processes, such as snowmelt, and evapotranspiration.

Keywords: solar radiation modelling; complex terrain; topography; remote sensing; GIS

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1. Introduction

Solar radiation is the ultimate energy source of the terrestrial system. It affects all the physical, chemical and biological processes on the Earth and sustains the living agents on the globe. In hydrology, the importance of solar radiation can be manifested by the fact that the spatially and temporally varying solar radiations regulate the spatial dynamics of many hydrological and landscape factors and processes, for example, temperature, photosynthesis and evapotranspiration, soil moisture and snow melting. These processes consequently affect the regional water balance (Fox et al. 2000), drought and flood occurrence (Marks et al. 2001), as well as landscape-scale distribution of vegetation (Urban et al. 2000, Dymond and Johnson 2002). Mapping of spatial solar radiation is an important prerequisite for many solar radiation-related applications. For locating solar energy system, monthly maps may be enough (Janjai et al. 2005), but for hydro-meteorological and hydro-ecological studies concerning rapidly changing processes, such as snowmelt, wind and vegetation growth, radiation variation at finer space and timescale is often required (Wang et al. 2002).

Solar radiation incident to the Earth’s surface consists of three components: direct radiation which is the radiation reaching the Earth’s surface without changing its direction; diffuse radiation which is the part of radiation scattered back to the Earth’s surface by atmosphere; and reflected radiation which is reflected by the surrounding terrain, amounting only a very small portion and is usually neglected in practice. The individual components as well as the sum of them, called global radiation, are determined by the interaction among Sun–Earth geometry (solar positioning, surface inclination, the terrain shading and so on), surface properties (albedo, absorptivity and so on) and atmospheric characteristics (atmospheric attenuation, cloud type and so on), all of which demonstrate a strong spatiotemporal variability. Solar radiation mapping may concern the individual radiation components (direct radiation, diffuse radiation or reflected radiation) or the total radiation amount (global radiation), under clear- or overcast-sky condition, on horizontal or inclined surface. Different approaches, that is, observations/interpolation, statistical methods, physical techniques or a combination of them have been developed to quantify the spatial radiation. Recent developments in remote sensing, such as the geostationary satellite data of METEOSAT (Cano et al. 1986, Rigollier et al. 2004), Geostationary Operational Environmental Satellite (GEOS) (Dubayah and Loechel 1997) and near-polar orbiting satellites, such as Moderate Resolution Imaging Spectroradiometer (MODIS) (Liang et al. 2006), offer new possibilities in solar radiation mapping. Mapping techniques also differ in terms of spatial and temporal scales.

This article first presents a review of the general steps of actual solar radiation modelling and the most popular modelling approaches for each step correspondingly. Unlike some other review articles focusing on detailed atmospheric processes or some single steps of the complete modelling chain, this article tries to present a roadmap of solar radiation modelling to address the demand of distributed hydrological modelling. Based on the review, two models with generally available data are selected and demonstrated comparatively for a small catchment in Southern Germany at different scales. One model is based on remote-sensing data from the geostationary meteorological satellite METEOSAT, called the Heliosat-2 model, and the other is based on easily observable daily sunshine duration observations, called Angström–Prescott (AP) method. Unlike some other researches focusing on the potential radiation (Kumar et al. 1997), this work addresses mainly the actual radiation under the influence of cloud and its spatiotemporal variability caused by topography. The objectives are (1) to quantify how strong the topography-induced spatial variability of solar radiation is; (2) to investigate how this variability changes with scale and...
identify a critical scale to capture the spatial variability between a balance of computational efforts and accuracy; and (3) to find operational radiation models driven by generally available data, which can be used to facilitate the development and application of distributed hydrological modelling.

2. Review of solar radiation modelling steps and alternatives

When it comes to the quantification of radiation, the difference between quantity and flux has to be noticed – following the terminology of WMO (2008), the term irradiation and radiation refer to the amount of energy falling on a unit area over a given time interval, while irradiance and radiance denote radiant flux of intensity, that is, the instantaneous solar power received by unit area per unit time. Radiation is also used as a generic term for solar radiant energy. Radiation can be measured by radiometers which are designed to be sensitive to a specified range of wavelengths (WMO 2008). The longwave radiation, direct radiation and global radiation can be measured by pyrgeometer, pyrheliometer and pyranometer, respectively. For an accurate solar radiation measurement, the devices have to be mounted according to the view-limiting geometry to capture exactly the desired radiation components with regular calibration and adjustment. The calibration of pyrgeometer also requires a lot of effort (Reda et al. 2003). The high requirements for calibration and maintenance render the radiation measurement network very sparse, for example, in Germany, altogether only 113 solar radiation stations are available, of which only 13 measure in addition the longwave radiation. Moreover, radiation measurement is very site specific, because of high spatial variability of ground surface and atmospheric conditions. It is therefore more usual to measure sunshine duration, \( n_d \), which is defined as the time period with direct solar irradiance exceeding 120 W m\(^{-2}\). The radiation pattern interpolated from the sparse radiation network can capture only spatial variability at the national or regional scale, but hydrological or hydro-ecological studies often require very detailed spatiotemporal radiation information; therefore, radiation modelling has been developed to meet this demand.

A good solar radiation model should be capable of handling arbitrarily oriented surfaces (horizontal and inclined) under all-sky conditions (clear sky and overcast sky) by properly representing and parameterizing the factors that affect the interactions among solar, atmosphere and ground surface. These factors include (Hofierka and Suri 2002) the following:

1. The Sun–Earth position: revolution and rotation.
2. Topography: elevation, surface aspect and inclination and shading.
3. Atmospheric characteristics: gases, water (liquid, solid, and vapour) and particles.

The first two groups of the above factors can be modelled by trigonometry with high level of accuracy. The atmospheric attenuation, which is the attenuation effect under clear-sky conditions, can only be modelled by parameterization with less but relatively good accuracy, because the composition of atmosphere is relatively stable. The most difficult issue is the cloud accounting; even the observed clouds data can be problematic due to rapid spatially and temporally changing weather conditions and the three-dimensional structure of cloud. Nevertheless, many models and software packages have been developed to model the global solar radiation, and some of them are included as a standard package in GIS software, for example, \( r.sun \) in GRASS (Hofierka and Suri 2002); the Solar Analyst (SA)
in ArcView and ArcGIS of ESRI (Fu and Rich 2002); and the SRAD in TAPEG (Wilson and Gallant 2000). Models differentiate themselves in terms of the approaches for geometric modelling, attenuation parameterization and overcast concepts, which will be discussed in the following sections.

2.1. Sun–Earth geometric modelling
The geometric modelling concerns the Sun’s position, solar incidence angle and shading. The basics of solar positioning algorithm and solar incidence angle calculation are very well known (see Iqbal 1983). In complex terrain, shading may result in significant difference of radiation, which can be caused either by the inclination of the surface itself or the adjacent terrain. Shading modifies the direct radiation by blocking the sunbeam. It also reduces the scope of the sky that scatters diffuse radiation to the ground. On the other hand, it may also add reflected radiation from the blocking terrain to the ground.

Two main shading algorithms are available: the solar-based method and the ground-based method. For the solar-based method, shading test is conducted for a given solar altitude at each time step for all points. The elevation of each ground point lying on the sun point line projection is checked to ensure that it does not obstruct the sunlight. The GRASS solar radiation package \( r \cdot s u n \) applies this method (Hofierka and Suri 2002). The ground-based method calculates the hemispherical viewing area (viewshed), which is the area that the point of interest is exposed to, based only on the adjacent terrain. Direct and diffuse radiation from this area is not shaded (Dozier and Frew 1990, Rich et al. 1994). Once the viewshed is constructed, at each time step no more computation but only a simple check whether the sun is lying within the viewshed is needed; therefore, the ground-based method is computationally more efficient. This method is represented by the SA package in ArcGIS and ArcView.

The geometric modelling gives the solar radiation in the absence of atmosphere on the ground surface. With the geometrical Sun–Earth relationship, the theoretical solar irradiance in the absence of Earth’s atmosphere can be calculated and integrated either numerically or analytically to get hourly or daily irradiation. The analytical solution of daily solar radiation and hourly solar radiation is shown in Iqbal (1983). But usually in the real world, the presence of air, trace gases, dust and water vapour in the atmosphere will attenuate and deplete the radiation, therefore, should be considered in application.

2.2. Atmospheric parameterization for clear sky
2.2.1. Fundamentals of atmospheric parameterization
The depletion processes include scattering by air molecules (Rayleigh scattering), scattering and absorption by aerosols, absorption by ozone layer, absorption by uniformly mixed gases and absorption by water vapour (Iqbal 1983). The total atmospheric attenuation effect is the result of interaction between atmosphere conditions and Sun–Earth revolution, because the optical air mass \( m_i \) of a given substance \( i \), which attenuates the incoming energy, changes with the solar ray direction and the richness of the substance in atmosphere. \( m_i \) can be expressed in relative terms with regard to Rayleigh optical air mass \( m_r \) as \( m_r (i) = m_i / m_r \). Attenuation processes can be described by Lambert’s law with two mutually convertible parameters, the transmittance \( \tau_i \) and the optical depth \( \delta_i \). Different substances scatter and/or absorb solar radiation at certain spectrum range and the attenuation processes for each substance and spectrum are independent, therefore, can be treated individually as monochromatic attenuation and then superimposed together (see Equations (1)–(3)).
\[ \dot{I}_i = \dot{I}_{0i} \tau_i \]  

\[ \tau_e = \tau_r \tau_a \tau_{oz} \tau_{gas} \tau_{wa} = \prod_{i=1}^{i=n} \tau_i \]  

\[ \tau_i = e^{-\delta_i} \]  

Here, \( i \) denotes a given atmospheric substance, which may or may not exist throughout the whole atmospheric layer. \( \dot{I}_i \) and \( \dot{I}_{0i} \) are the intensities of spectral radiation before and after passing through the atmosphere. \( \tau_e, \tau_r, \tau_a, \tau_{oz}, \tau_{gas}, \tau_{wa} \) are the effective transmittance, transmittance of Rayleigh (molecule) scattering, transmittance of aerosol scattering and absorption, transmittance of ozone absorption, transmittance of uniformly mixed gas and transmittance of water vapour, respectively. The optical depth \( \delta_i \) is substance specific, which is different from the optical path length determined only by incoming light angle. Part of the attenuated radiation becomes diffuse radiation, which can be conceptualized by the diffuse transmission coefficient \( T_d \).

Numerous atmospheric parameterization schemes exist (Ineichen 2006), which can be generalized into two groups based on whether the attenuation effects of different extinction substances are parameterized substance specifically or integrated. The substance-specific parameterization adopts different attenuation parameters for each individual extinction layer (substances), which is represented by the Page (1997) model, SOLIS model (Mueller et al. 2004), Bird and Huldstrom (1980) model and CPCR2 model (Gueymard 1993). The second group summarizes the different processes into Rayleigh optical depth \( \delta_r \) and the Linke turbidity \( T_{Ln} \), by which the transmittance of each extinction substance is converted and summed up with reference to the spectrally integrated Rayleigh optical depth of the clean and dry atmosphere (Kasten 1996). It is used by the European Solar Radiation Atlas model (Rigollier et al. 2000) and the Ineichen model (Ineichen and Perez 2002). Linke turbidity can be derived by measurements of the beam irradiance using appropriate but expensive equipment. The typical Linke turbidity values are widely reported in the literature for different parts of the world.

### 2.2.2. Clear-sky model on horizontal surface

Based on the parameterization schemes, the clear-sky irradiance, both direct and diffuse components on horizontal surface, can be expressed as the following in integrated and substance-specific form, respectively.

- **Direct radiance**

  Integrated parameterization:
  \[ b_{hc} = \dot{I}_{SC} \cos \theta_z \exp(-0.8662 T_{Ln} m_r \delta_r(m_r)) \]

  Substrate-specific parameterization:
  \[ b_{hc} = \dot{I}_{SC} \cos \theta_z \tau_r \tau_a \tau_{oz} \tau_{gas} \tau_{wa} \]
• Diffuse irradiance
Integrated parameterization:

\[ d_{hc} = \dot{I}_{SC} \in T_d(T_{Ln})F_d(\theta_z,T_{Ln}) \]  

Substance-specific parameterization:

\[ d_{hc} = \dot{I}_{SC} \in T_d(\tau_e,f_{as},f_{af}) \]  

Here, the subscript h denotes horizontal surface and c denotes clear sky. \( \dot{I}_{SC} \) is the extraterrestrial solar radiation. \( \theta_z \) is the solar zenith angle of horizontal surface and \( \theta \) is used to denote the zenith angle of inclined surface. \( \epsilon \) is the eccentricity correction factor of Sun–Earth distance. The diffuse transmission coefficient \( T_d \) for the integrated parameterization is a function of \( T_{Ln} \), and for individual parameterization it is given by Equation (7). The integrated form also includes a diffuse angular function \( F_d \) related to \( \theta_z \) and \( T_{Ln} \). Here \( f_{af} \) denotes the portion of forward scattered radiation and \( f_{as} \) is the scattered fraction out of the total energy attenuated by aerosol.

2.2.3. Clear-sky model on inclined surface

For inclined surface under clear sky, the direct radiation will be modified by the solar incidence angle (see Equation (8)), and the diffuse radiation will be reduced in addition by the reduced skyview fraction \( \psi_\beta \), also called the diffuse coefficient (see Equation (8)):

\[ b_{\beta c} = b_{hc} \cos \theta / \cos \theta_z \]  
\[ d_{\beta c} = d_{hc} \psi_\beta \]  

Here the subscript \( \beta \) denotes inclined surface. The relationship between inclined and horizontal surface under clear sky also holds for cloudy condition. The formulation of the coefficient \( \psi_\beta \) depends on how the diffuse radiation is treated. The basic component of diffuse radiation is the isotropic part from all directions of the sky, sometimes called dome or sky irradiation. In addition, two anisotropic components may be considered – circumsolar brightening caused by the strong forward aerosol scattering from approximately 5° around the direct solar beam, and the horizon brightening due primarily to multiple Rayleigh scattering and retroscattering in clear atmospheres. The diffuse radiation model may consider only the isotropic component as by Liu and Jordan (1961), or in addition the circumsolar component, for example, by Hay (1979), or eventually all the three components, that is, by Perez et al. (1990) and Muneer (1990). There is an array of diffuse radiation models which will not be repeated here. These models are applicable for both clear sky and overcast conditions. For a complete overview of them, readers are referred to the article of Evseev and Kudish (2009). Here the Liu–Jordan model and the Muneer model, which are components in the selected models of this article are given briefly as follows:

(1) Liu–Jordan (1961) model:

\[ \psi_\beta = \cos^2(\beta/2) = (1 + \cos \beta)/2 \]
(2) Muneer (1990) model:

\[
\psi_\beta = T_M (1 - F_M) + F_M r_s \tag{11}
\]

\[
T_M = \cos^2(\beta/2) + U \left[ \sin \beta - \beta \cos \beta - \pi \sin^2(\beta/2) \right] \tag{12}
\]

\[
U = 0.00263 - 0.712F_M - 0.688F_M^2 \tag{13}
\]

\[
r_s = \max[0, (\cos \theta / \cos \theta_z)] \tag{14}
\]

\[
F_M = K_{hay} \text{ for sunlit surface and non-overcast sky} \tag{15}
\]

\[
F_M = 0 \text{ for surface in shadow} \tag{16}
\]

\[
K_{hay} = b_{hc} / I_{0h} \tag{17}
\]

Here, \( \beta \) is the slope of the inclined surface; \( K_{hay} \) is the Hay’s sky clarity index (Hay 1979) denoting the proportion of beam irradiance and extraterrestrial solar irradiance on horizontal surface; \( T_M \) is the Muneer’s tilt factor; \( F_M \) is a composite clearness function; and \( r_s \) and \( U \) are some auxiliary variables.

### 2.3. Cloud accounting for cloudy sky

Cloud is the most critical factor for calculating the actual solar radiation. It is also the most uncertain factor due to its temporal and three-dimensional spatial variability. Actual radiation under overcast sky is usually processed step by step: first the overcast global radiation on horizontal surface \( G_{hw} \) is obtained by modelling or observation, then the estimation of direct and diffuse radiation proportion by cloud parameterization, finally the corresponding direct and diffuse solar radiation on the inclined surface, \( B_{\beta w} \) and \( D_{\beta w} \), will be estimated based on their corresponding horizontal values, \( B_{hw} \) and \( D_{hw} \), and summed up to get the global radiation on the inclined surface \( G_{\beta w} \).

#### 2.3.1. Overcast global radiation on horizontal surface

Because it is impossible to consider the instantaneous change of cloud, cloud is usually parameterized in a time averaging way at hourly or daily interval. Detailed physical modelling of cloud field for radiation analysis is beyond the objective of this work or even beyond the demand of hydrological study. Cloud effects are usually quantified by a parameter called clear-sky index, \( K_w \) (Kasten and Czeplak 1980), which is the ratio of actual global radiation to the potential global radiation on horizontal surface under clear-sky conditions (Equation (18)). \( K_w^b \) and \( K_w^d \) are the respective beam and diffuse components of the clear-sky index (Equations (19) and (20)). It has to be distinguished from other similar parameters: (1) clearness index \( K_t \) in Equation (21) which is the ratio of global radiation on the horizontal surface \( G_{hb} \) to extraterrestrial radiation on horizontal surface \( H_0 \) and (2) the Hay’s sky clarity index \( H_{hay} \) mentioned previously.

\[
K_w = G_{hw} / G_{hc} \tag{18}
\]
The clear-sky index can be interpreted as the cloud transmittance and it can be obtained, as its definition suggests, as the ratio between the observed $G_{hw}$ and the theoretically derived $G_{hc}$. When no radiation observation is available, $K_w$ is expressed as an empirical function of one of the following parameters: (1) the sunshine duration, (2) the cloudiness index, (3) the cloud index or (4) some combination of meteorological parameters, such as, precipitation, temperature and humidity, etc.

- The sunshine duration method

AP equation (Angström 1924) has suggested that the monthly average daily global radiation on a horizontal surface can be estimated through the sunshine duration as follows:

$$\frac{G_{hw}}{G_{hc}} = K_w = p_{ap} + q_{ap} n_d / N_{dh} \tag{22}$$

Here, $p_{ap}$ and $q_{ap}$ are the empirical coefficients, and $n_d$ and $N_{dh}$ are the actual and potential sunshine durations, respectively. Although the original form is proposed for the evaluation of the long-term average daily global radiation, it is proved to be held for daily radiation as well. There are also many non-linear versions of AP equation which will not be listed here.

- The cloudiness index method

The cloudiness index $C_k$ is the spatial cloud cover in okta unit. It does not consider the cloud type or thickness. Kasten (1983) proposed the following relationship between cloud cover and clear sky index, where $p_k$ and $q_k$ are empirical coefficients:

$$K_w = 1 - \left( p_k \left( \frac{C_k}{8} \right) \right)^{q_k} \tag{23}$$

- The meteorological method

The cloud index $n_c$ is defined specifically for each pixel based on albedo data derived from the visible (VIS spectrum range 0.5–0.9 μm) band images of the satellite remote-sensing data, for example, METEOSAT. Here, $K_w$ is a function of $n_c$ (Cano et al. 1986, Rigollier et al. 2004).

$$n_c = (\rho_o - \rho_g) / (\rho_a - \rho_g) \tag{24}$$

$$K_w = f(n_c) \tag{25}$$
where $\rho_o$ is the apparent albedo observed by the satellite sensor [−]; $\rho_g$ is the apparent albedo of the ground under clear skies [−]; and $\rho_a$ is the transmittance of aerosol scattering and absorption [−].

Albedo increases with the cloud cover, which is bright and has higher reflection. $\rho_g$ and $\rho_a$ are the minimum and maximum that are selected from the time series of satellite images at each pixel. This is the method that is adopted in the Heliosat-2 method (Rigollier et al. 2004). Actually with the development of meteorological satellites, an alternative of cloud parameterization that does not belong to the clearness index family called cloud optical depth has emerged (Mueller et al. 2004) but is still subject to improvement.

Once the clear-sky index, $K_w$, is known, the overcast global radiation on horizontal surface $G_{hw}$ can be obtained by Equation (18), given that $G_{hc}$ is available from observation or model estimation. Then using Equation (21), the clearness index $K_t$ can be obtained for further calculation. It should be mentioned that for some methods the calculation order of $K_t$ and $G_{hw}$ is inversed – first $K_t$ is directly regressed with sunshine duration $n_d$ (Prescott 1940, Bahel et al. 1987) or meteorological parameters, such as temperature (Bristow and Campbell 1984, Allen 1997), then based on $K_t$, $G_{hw}$ is calculated. The latter method is supposed to be rougher than the former one, because no physical processes are explicitly considered.

2.3.2. Radiation partitioning on the horizontal surface

For further estimation of diffuse and direct radiation on inclined surface under cloudy conditions, the beam and diffuse component under cloudy conditions on the horizontal surface have to be partitioned. They are usually estimated by statistical regression of observed data, which relate either the direct radiation fraction, usually called direct radiation transmittance $K_b$ (see Equation (26)), or the diffuse radiation fraction $K_d$ (see Equation (27)) to the clearness index, the surface albedo, the sun elevation (or say relative air mass) or some other parameters, such as dew point temperature and humidity. If the reflected radiation is neglected, $K_b$ and $K_d$ sum to unity, that is, Equation (28) holds.

\[ K_b = \frac{B_{hw}}{G_{hw}} \]  \hspace{1cm} (26)

\[ K_d = \frac{D_{hw}}{G_{hw}} \]  \hspace{1cm} (27)

\[ K_b + K_d = 1 \]  \hspace{1cm} (28)

• Direct radiation transmittance $K_b$

The DISC model of Maxwell (1987) first proposed the expression of direct radiation transmittance (normal to the incident surface) as the function of clearness index and relative air mass using regression analysis, but with some physical considerations. Perez et al. (1992) further developed the model into the Drint model, which considers more parameters, such as precipitable water. DirIndex model (Perez et al. 2002) is a further update of Drint and can be generalized as follows:

\[ K_b = f(K_t, n_d, \ldots) \]  \hspace{1cm} (29)

• Diffuse radiation fraction $K_d$
Diffuse radiation fraction $K_d$ is a more popular approach in application which has a longer history. It is expressed as a polynomial function of clearness index (Liu and Jordan 1961, Ruth and Chant 1976) or the sunshine duration (Iqbal 1983). Logistic functions (Boland et al. 2001) and functions with multiple predictors (Ridley et al. 2010) have also been developed to include more parameters, such as temperature, humidity and turbidity, as a correction of the clearness index or the sunshine duration. Despite the functional form, all these equations are mainly regression analysis based on observed data (see Equation (30)), even though in some cases physical processes may be incorporated. Comparative studies on diffuse radiation modelling can be found in Torres et al. (2010).

$$K_d = f(K_t, n_d, \ldots)$$ (30)

2.3.3. Global radiation on inclined surface

Based on the partition factors, the direct and diffuse radiation on horizontal surface can be obtained. Through a procedure similar to the clear-sky conditions, direct and diffuse irradiance on inclined surface can be calculated following Equations (8) and (9) by replacing the subscript $c$ with $w$. The instantaneous values should also be integrated analytically or numerically to obtain the corresponding radiation of a given time interval. To give a better overview of the global solar radiation modelling, we summarize the modelling steps and options in a pseudo flowchart (see Figure 1), which gives the main steps and alternatives of solar radiation modelling. Models generally comprise the following steps: horizontal extraterrestrial radiation $H_0 \rightarrow$ clear sky global radiation on horizontal surface $G_{hc} \rightarrow$ overcast global radiation on horizontal surface $G_{hw} \rightarrow$ radiation partition $\rightarrow$ overcast global radiation on tilted surface $G_{\beta w}$. For each step, different alternatives discussed in the previous sections can be chosen depending on the data availability.

3. The Heliosat-2 and AP models

As discussed above, atmospheric characterization and cloud parameterization are the most critical factors for calculating potential radiation and actual radiation, respectively. For a given region, clear-sky atmospheric conditions are relatively stable and can be easily characterized as transmittance or Linke turbidity with 1-year historical measurement data or even data obtained from other similar geographic regions. Since cloud is more dynamic and site specific, different atmospheric observations are required to parameterize the cloud effects, which require considerable observatory efforts. Therefore, easily attainable data such as sunshine duration which does not require a complicated instrument, or generally available data such as remote sensing, should be the first option for the modelling purpose. Cloud parameterization using remote-sensing data is an ideal substitution of ground-based data. Solar radiation model based on MODIS, GEOS and METEOSAT data has shown very promising performance in some prior studies (Dubayah and Loechel 1997, Hammer 2003, Liang et al. 2006). In this article, two cloud parameterization schemes, namely, physically parameterized, remote-sensing-oriented model Heliosat-2 (Rigollier et al. 2004) and the regressional AP model based on sunshine duration are selected and compared. Both models are representative models of their type and have a balanced complexity in the model components. The Heliosat-2 method applies the $r.sun$ (Hofierka and Suri 2002) approach together with a specific cloud parameterization scheme developed by European Solar Radiation Atlas (Rigollier et al. 2000). The AP method applies sunshine duration as
There are many comparative studies for point or potential solar radiation with the aforementioned methods (Muneer and Gul 2000, Iziomon and Mayer 2002, Trnka et al. 2005, Ineichen 2006, Wu et al. 2007, Evseev and Kudish 2009, Torres et al. 2010), and some of them address specially GIS solar radiation modelling software packages (Ruiz-Arias et al. 2009). GIS-based models calibrated with station data have been widely applied to

a surrogate for the total attenuation effects of both cloud and atmosphere. Both models are implemented in a GIS framework in this work.
model the spatial solar radiation (Mckenney 1999, Wang et al. 2006, Batlles et al. 2008, Martnez-Durbn et al. 2009, Ruiz-Arias et al. 2009), but these methods always require station observed radiation data, which limit the transferability of these models in ungauged basins, or they calculate only potential values. Approaches for modelling spatial actual solar radiation in complex terrain with generally available data for hydrological applications are investigated in this article, with the focus on the issue of topography-induced spatial radiation variability.

3.1. The Heliosat-2 model

For the Heliosat-2 method, first, the clear-sky index is obtained from the cloud index \( n_c \), then the overcast global radiation on horizontal surface \( G_{hw} \) is calculated by Equation (18), which can be subsequently used to derive the clearness index \( K_t \). The diffuse radiation fraction \( K_d \) is estimated as a function of \( K_t \) to calculate diffuse radiation on horizontal surface, based on which the beam and diffuse components of the clear-sky index, \( K_{b, hw} \) and \( K_{d, hw} \), can be calculated for spatial radiation mapping. \( K_{b, hw} \) and \( K_{d, hw} \) are assumed to be constant throughout the day. The Muneer’s model is applied to obtain the diffuse radiation on inclined surface, that is, to calculate \( \psi_{\beta} \). The global radiation on an inclined surface at a grid point \( p(i, j) \) can be calculated as follows:

\[
G_{\beta, w}(i, j) = B_{\beta, w}(i, j) + D_{\beta, w}(i, j)
\]

\[
= \int_{\omega_{sr}}^{\omega_{ss}} (b_{\beta, w}(i, j) + d_{\beta, w}(i, j))d\omega
\]

\[
= \int_{\omega_{sr}}^{\omega_{ss}} (b_{hw}(i, j) \cos \theta / \cos \theta_z + d_{hw}(i, j)\psi_{\beta})d\omega
\]

\[
= \int_{\omega_{sr}}^{\omega_{ss}} (b_{hc}(i, j)K_{b, hw}(i, j) \cos \theta / \cos \theta_z + d_{hc}(i, j)K_{d, hw}(i, j)\psi_{\beta})d\omega
\]

Here, \( \omega \) is the solar hour angle, with \( \omega_{sr} \) being the sunrise hour angle and \( \omega_{ss} \) the sunset hour angle. The geolatitude and sky condition can be assumed to be constant within the catchment. Therefore,

\[
K_{b, hw}(i, j) = K_b^c \quad \forall i, j
\]

\[
K_{d, hw}(i, j) = K_d^c
\]

Eventually, Equation (31) can be simplified in the following form:

\[
G_{\beta, w}(i, j) = \int_{\omega_{sr}}^{\omega_{ss}} (b_{hc}K_{b, hw}^c \cos \theta / \cos \theta_z + d_{hc}K_{d, hw}^d \psi_{\beta})d\omega
\]
3.2. **The AP model**

The AP model applies the relative sunshine duration $n_c$ as a surrogate for the attenuation effects of both cloud and atmosphere. The diffuse radiation fraction $K_d$ is approximated as a function of $n_d$. For the diffuse radiation on inclined surface, the Liu–Jordan model in Equation (10) is applied, because it is conceptually much simpler, which matches the degree of complexity of the AP method. The Liu–Jordan method estimates only isotropic diffuse component, which is constant over time, by geometric calculation of the sky exposure of the tilted surface. It also neglects the shading effect for direct radiation. The hourly value is integrated analytically to get the daily value by assuming constant diffuse fraction throughout the day. Following similar procedure for extraterrestrial radiation in Iqbal (1983), Equation (34) can be obtained as

$$G_{\beta w}(i,j) = \int_{\Omega_{\beta w}}(b_{hw}(i,j) \cos \theta / \cos \theta_z + d_{hw}(i,j) \psi_{\beta})d\omega$$

(34)

The overcast global radiation on horizontal surface can be calculated with an adapted AP model relating $G_{hw}$ directly to extraterrestrial radiation $H_0$:

$$G_{\beta w}(i,j) = G_{hw}(1 - K_d) \xi + G_{hw} K_d \psi_{\beta}$$

$$= H_0(a_{ap} + b_{ap} n_d / N_{dh})(1 - K_d) \xi + H_0(a_{ap} + b_{ap} n_d / N_{dh}) K_d \psi_{\beta}$$

(35)

Here, $\xi$ is the ratio of daily radiation on inclined surface to horizontal surface in the absence of the atmosphere. The readers are referred to Iqbal (1983) for a detailed derivation.

4. **Comparison of the selected models**

At the first step the geometrical modelling of $r.sun$ will be checked against observations and another widely used model SA on horizontal surface under clear-sky conditions. Then the scale sensitivity of $r.sun$ will be tested with 30 to 500 m DEM resolutions. After the clear-sky test, the performance control of the Heliosat-2 and AP models under cloudy conditions will be conducted based on global and diffuse radiation measured on horizontal surface at stations. Because solar radiation observation data for this study are only for horizontal surfaces, the two models are tested against each other for inclined surfaces. Finally the regional radiation mapping, that is, topographic downscaling of solar radiation using two methods will be performed.

4.1. **Test area**

The solar radiation stations in German federal state Baden-Wuerttemberg (see Figure 2) are used to test the Heliosat-2 and AP models. There are three solar radiation stations within the state – Mannheim, Stuttgart and Freiburg, which provide daily sunshine duration, global and diffuse solar radiation data from 2002 to 2007. For Stuttgart station global radiation data from year 1990 to 1999 are also available. A small catchment within the state
called Talhausen close to Stuttgart city is used to test the models on horizontal surface. The Talhausen catchment is around 192 km², with elevation varying from 216 to 530 m for a 30 m DEM. The southern part of the catchment is a mountainous area with steep valleys, while the north part is relatively flat, which forms a complex terrain with diversified topographic features. Four locations, P1–P4, with representative topographic features, are marked for detailed investigation. The selected points are around 10–15 km away from the Stuttgart solar station, which is close enough to allow a parameter transfer. Table 1 shows the topographic parameters (elevation, aspect, slope) of the four points at DEM resolution from 30 to 500 m. In general, elevation and slope decrease with increasing grid size, whereas the variation of aspect is strongly dependent on the surrounding terrain. However, due to the resampling procedure of DEM data processing, some erratic variations of these parameters may occur, for example, instead of monotonically decreasing, the elevation of the selected points fluctuates.

4.2. Test of geometric modelling

As discussed before, \textit{r.sun} applies the solar-based shading algorithm, and it is conceptually simpler than the hemispheric viewshed algorithm used in \textit{SA}, which is assumed to be more advanced and computationally efficient. In this article, the \textit{r.sun} program incorporated in \textit{GRASS} is adapted to be a stand-alone program for more flexible calculations.

4.2.1. Comparison of point radiation

The performances of the two algorithms are compared using simulation of time series at P1 under clear-sky conditions. The observed solar radiation at Stuttgart station is included as a reference case. Figure 3 shows the results of the eight different cases at P1 as follows:
Table 1. Topographic features of P1–P4 at different resolutions.

<table>
<thead>
<tr>
<th>DEM resolution</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elevation (m)</td>
<td>Aspect (°)</td>
<td>Slope (°)</td>
<td>Elevation (m)</td>
</tr>
<tr>
<td>30 m</td>
<td>416</td>
<td>7.91</td>
<td>16.85</td>
<td>264</td>
</tr>
<tr>
<td>60 m</td>
<td>422</td>
<td>14.12</td>
<td>18.41</td>
<td>264</td>
</tr>
<tr>
<td>100 m</td>
<td>424</td>
<td>21.07</td>
<td>16.27</td>
<td>265</td>
</tr>
<tr>
<td>150 m</td>
<td>429</td>
<td>20.04</td>
<td>14.71</td>
<td>260</td>
</tr>
<tr>
<td>300 m</td>
<td>404</td>
<td>22.91</td>
<td>9.95</td>
<td>246</td>
</tr>
<tr>
<td>500 m</td>
<td>399</td>
<td>29.14</td>
<td>6.93</td>
<td>245</td>
</tr>
</tbody>
</table>

Figure 3. Observed and simulated global radiation by \( r.\text{sun} \) and Solar Analyst (SA) for different cases at P1.

1. Maximum daily global radiation observed at station Stuttgart for all data available years (blue line with circles): the maximum daily radiation over 18 years is taken to approximate the clear-sky radiation.
2. SA-simulated clear-sky radiation on horizontal surface assuming flat DEM (green line): clear-sky parameters of SA with transmittance \( \tau_e \) equal to 0.75 and diffuse portion \( K_d \) equal to 0.2 obtained by trial and error can best approximate case 1.
3. SA-simulated clear-sky radiation on horizontal surface with actual DEM for surrounding area (orange line): the result shows the shading effect of the surrounding terrain by SA.
4. SA-simulated clear-sky radiation on inclined surface with actual DEM (purple line): the result shows the shading effect of both surrounding terrain and the inclined surface itself.
5. \( r.\text{sun} \)-simulated clear-sky radiation on horizontal surface without shading effect (grey line): the Linke turbidity \( T_{ln} \) varies from 3.0 in December to 4.6 in winter, as suggested by Kasten and Czeplak (1980).
6. \( r.\text{sun} \)-simulated clear-sky radiation on horizontal surface with shading effect (brownish line).
In general, both the simulated global radiation (Figure 3) and sunshine duration (Figure 4) in different cases from the two models are consistent. Case 1 of both figures show that the profiles of the maximum daily curves, both global radiation and sunshine duration, resemble the theoretical cosine curves under clear-sky conditions very well. Clear-sky radiation simulated by SA (case 2 in Figure 3) on horizontal surface shows an exaggerated difference from case 1 for summer time and has difficulties in matching the observed curve for both summer and winter seasons. The corresponding sunshine duration (case 2 in Figure 4) shows a stepwise feature and exceptional fluctuations in January and December. A better match may be realized by a time-variant parameter setting which is not possible in the SA module.

If shading from surrounding terrain is considered for horizontal surface, as shown in Figure 3, both SA and r.sun (cases 3 and 6) give much lower radiation for winter time compared with the no shading case and little difference in summer (cases 2 and 5). Comparison of all the shade/no shade cases in Figure 4 shows that in summer time illumination time is identical, because P1 lies on a north-oriented slope and shading happens only in winter time when the sun altitude is low. The inclined surface receives much less radiation than the horizontal surface, which is represented by both models. For P1, the sunshine durations for horizontal surface and inclined surface calculated by r.sun, under the presence of shading are identical (cases 6 and 8 in Figure 4), which means shading is caused by the neighbouring terrain, not by the inclination of the surface itself. But this is not true for SA, which shows slight difference (cases 3 and 4 in Figure 4) possibly arising from the accounting of detailed microtopography by SA. However, when the surface is tilted, the inclination will be responsible for the shading effect in summer time. Even though shading reduces more illumination time for inclined surface in winter comparing to summer (cases 7 and 8 in Figure 4), it makes little difference in global radiation (cases 7 and 8 in Figure 3). The reason may lie in that shading happens at sunrise or sunset time, for which the solar incidence angle is very small due to the surface inclination, and the reduced beam radiation is almost

Figure 4. Observed and simulated sunshine duration by r.sun and Solar Analyst (SA) for different cases at P1.
compensated by the increased diffuse radiation due to horizontal brightening. SA and \textit{r.sun} show better agreement for inclined surface than for horizontal surface.

The basic validation against observation has shown that both models, especially \textit{r.sun}, perform reasonably well on horizontal surface under clear-sky conditions, and for inclined surface the two models also give consistent results with minor differences. The performance of SA is not so good as \textit{r.sun} with time-varying parameters, but still reasonable. SA is specially error prone for the equinox and solstice days, which can be witnessed by the unusual values on 22 December of the green curves. The agreement of the two models on inclined surface can serve as an indirect verification of the models on inclined surface, given that observed data for inclined surface are not available.

4.2.2. Comparison of spatial radiation

The spatial comparison of \textit{r.sun} with SA is conducted for an arbitrarily selected day (as an example day 30 is selected, see Figure 5a and b) and the yearly average daily radiation (see Figure 5c and d). The scatter plot shows that there is a high correlation between the two models for the simulated radiation and the sunshine duration, except for very few outliers. The \textit{r.sun} model gives in general slightly higher evaluation than SA for global radiation, mainly in the valleys. The reason may lie in the fact that SA considers the microtopography in more detail. For extremely low sunshine duration values, which are points in shadow, \textit{r.sun} gives lower values than SA. Nevertheless, the strong linear relationship between the two model results implies that by adjusting the parameters properly, a good match of the two models is possible. For the yearly average daily radiation, the difference diminishes, and results of the two models become more close as shown in Figure 5c and d.

4.2.3. Correlation of radiation with topography

Statistical analysis shows that there is a positive correlation of radiation with aspect and elevation, and negative correlation with slope. Table 2 shows the mean of daily correlation coefficients and standard deviation over 1 year. The aspect starts at north zero clockwise. It has to be mentioned that in GRASS open GIS system, the aspect increases counterclockwise, with east as 0°. To consider the topography more properly, the negative cosine of aspect is taken to calculate the correlation coefficient. Similarly, the sine of slope is used, and only elevation uses the original value. The correlation between elevation and global radiation is very weak, but an interesting time-dependent correlation can be observed, namely, the correlation coefficient is negative in winter and positive in summer.

4.3. Test of scale sensitivity

In GIS-based models, the calculation of radiation is based on DEM data; therefore, it is natural that the resolution of the DEM affects the simulated radiation by modifying the topographic parameters and shading conditions. In the following, how the shading effect and spatial variability of simulated radiation changes with scale is investigated with DEM resolutions ranging from 30 m to 500 m.

4.3.1. Shading effect with scale

Shading reduces the global radiation of one point in shadow (from \( G_{ns} \) to \( G_s \)). When the DEM grid size increases, the microtopographic features which can be reflected at finer resolution disappear, which causes some information to be lost and thus affects the shading...
Figure 5. Scatter plot of $r_{sun}$-simulated and Solar Analyst (SA)-simulated spatial radiation and sunshine duration. (a) Scatter plot of solar radiation on day 30 and (b) the corresponding sunshine duration. (c) Daily average solar radiation over 1 year and (d) the corresponding sunshine duration.

Table 2. Correlation between the terrain parameters and global radiation.

<table>
<thead>
<tr>
<th></th>
<th>Aspect</th>
<th>Slope</th>
<th>Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean correlation coefficient over 365 days (year 2000)</td>
<td>0.773</td>
<td>-0.416</td>
<td>0.017</td>
</tr>
<tr>
<td>Standard deviation of correlation coefficients</td>
<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
</tr>
</tbody>
</table>

effect, for example, two points with one being shaded by the other may be merged as one point in a coarser resolution DEM, or the shading time may change at different scales. Here, the annual mean daily radiation simulated with and without shading effect are compared for six different DEM resolutions. The cumulative distribution of the relative difference ($G_{ns} - G_s$) of all grid points of each DEM resolution is demonstrated in Figure 6. It shows that the shading effect prevails at fine resolution. But even at a resolution of 30 m, only in about less than 5% area shading causes a relative difference larger than 5%. When the grid size increases, the shading effect drops quickly below 5% in most of the area. It can be concluded that the shading effect is only detectable locally for very limited area, which may be significant for some ecological applications, but not hydrological applications at catchment level.
4.3.2. Scale effect over time

To check the scale effect over time, two pairs of points, P1 and P2 in mountain area and P3 and P4 in lowland area, are examined without considering the shading effect. The results show that the radiation difference varies with scale (see Figure 7). In general, both absolute and relative differences diminish with increasing grid size – for P1 and P2, the relative difference decreases from around 600% at 30 m DEM to less than 60% at 500 m DEM, and for P3 and P4 from 80% to around 1%. For P3 and P4, due to the shift of aspect at P3 from north/north-east to south-east and at P4 from south-west to north-west when transiting from 60 m DEM to coarser DEM, the relationship at the two points gets inverted. This shows that DEM-based radiation modelling may be erratic at some points, but at the basin scale, such a phenomenon should be of minor importance. For both pairs, the scale effects in terms of absolute radiation are the largest at the equinox days, and decrease on both solstice days, with a minimum difference in the day of the summer solstice. However, if relative radiation is considered, the scale effect is much stronger in winter than in summer.

4.3.3. Spatial radiation variance with scale

The spatial variation of solar radiation is quantified in terms of normalized interquantile range (NIQR) \( P_{90} - P_{10} / \mu_s \), where \( P_{90} \) and \( P_{10} \) are the 90 and 10 percentiles of the spatial values in a specific radiation map, and \( \mu_s \) is the mean radiation of that map. We do not use the standard deviation, because the distribution of spatial radiation depends on the topography, which is not necessarily normally distributed. Figure 8 shows that in winter the spatial variability of global radiation is very sensitive to the scale and decreases from 37% for 30 m DEM to 19% for 500 m DEM, especially when the grid size increases further to 100 m, the spatial variability drops rapidly, whereas in summer time the spatial variability is below 5% and does not vary much with scale. This may have strong implication for the energy-driven hydrological processes in the winter time, such as snowmelt and evaporation, and solar radiation modelling at resolution finer than certain critical value is preferred for distributed hydrological modelling to account for such spatial variability explicitly.
4.4. Performance test on overcast horizontal surface

For testing the model performance on all-sky conditions, the Heliosat-2 model is validated against the station data at Stuttgart, Mannheim and Freiburg from year 2002 to 2007. The adapted AP method using extraterrestrial radiation is also tested against the observations.

4.4.1. The Heliosat-2 method

The Heliosat-2 method calculates the cloud index using Equation (24). The functional form to derive clear-sky index $K_w$ from the cloud index $n_c$ (Equation (25)) follows Rigollier et al. (2004):

$$K_w = \begin{cases} 
1.2 & n_c < -0.2 \\
1 - n_c & -0.2 \leq n_c \leq 0.8 \\
2.0667 - 3.6667n_c + 1.6667n_c^2, & 0.8 \leq n_c \leq 1.1 \\
0.05 & n_c \geq 1.1
\end{cases}$$

(36)
The overcast radiation $G_w$ is equal to the clear-sky radiation $G_{hc}$ scaled by the clear-sky index $K_w$. Table 3 shows the simulated result in comparison with observations at the three stations. The goodness of fit is tested by the root mean squared error (RMSE) and $R^2$. The $R^2$ for all the three stations are above 0.93, which shows a very good match between simulation and observation.

4.4.2. The AP method

The original AP (Equation (22)) can be adapted to the following form using extraterrestrial radiation instead of clear-sky radiation to obtain the clearness index $K_t$:

$$\frac{G_w}{H_0} = K_t = p_{ap} + q_{ap}n_d/N_{dh}$$  \hspace{1cm} (37)
The actual daily sunshine duration \(n_d\) is obtained from the observations, while the potential daily sunshine duration and extraterrestrial radiation on horizontal surface, \(N_{dh}\) and \(H_0\) can be easily calculated following Iqbal (1983). To apply the AP equation, \(\alpha_{ap}\) and \(\beta_{ap}\) have to be estimated first. The observation data are split into two time series of equal length, with 2002–2004 for calibration and 2005–2007 for validation. It shows that \(\alpha_{ap}\) increases and \(\beta_{ap}\) decreases with the latitude (see Table 3), which are consistent with the findings of Glover and McCulloch (1958) and can be regarded as a kind of climatic gradient. Because in our case, the three solar radiation stations are relatively close, a single set of AP coefficients can be used, which is called the ‘general’ case. It is not surprising that the calibrated empirical AP approach is slightly better than the physically based Heliosat-2 method. But one should keep in mind that the Heliosat-2 method is free of any ground observations, which offers a satisfactory calibration-free solution for ungauged basins.

Because the radiometers for routine measurements of solar radiation are usually mounted horizontally, ground sources for validation of radiation on tilted surface are very scarce. A model test for the inclined surface is not possible due to the data limitation in the study region. Therefore, the capability of the two methods for modelling tilted surface under overcast sky condition is verified by comparing against each other.

### 4.5. Spatial solar radiation mapping

#### 4.5.1. Derivation of diffuse fraction

For the Heliosat-2 method, a ground observation-free method following the Ruth–Chant approach (Ruth and Chant 1976) has been applied to relate diffuse fraction to clearness index in the following form:

\[
K_d = \begin{cases} 
K'_d, & K_t \leq K'_t \\
\frac{p_1(K_t)^2 + p_2k_t + p_3}{k_t}, & k_t \geq k'_t
\end{cases}
\]  

(38)

Here, \(K'_t\) is the threshold value of clearness index for applying the quadratic function, and \(p_1, p_2\) and \(p_3\) are empirical coefficients. The clearness index is obtained from the global radiation on horizontal surface. Once the direct and diffuse components on horizontal surface are partitioned with \(K_d\), the direct and diffuse components of the clear-sky index, \(K_w^b\) and \(K_w^d\), are calculated for further calculations.

For the AP method, the diffuse fraction is evaluated as a quadratic function of relative sunshine duration:

\[
K_d = p'_1(n_{rel})^2 + p'_2n_{rel} + p'_3
\]  

(39)

Tables 4 and 5 show the calibration and validation results of each station with the clearness index method and sunshine duration method, respectively. Both functional forms are calibrated and validated with two approaches:

2. General method: calibration with data from all three stations and validation for each station for the validation period.
Table 4. Diffuse fraction obtained from clearness index method for the Heliosat-2 model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Calibration</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_d$</td>
<td>$K_t$</td>
<td>$p_1$</td>
</tr>
<tr>
<td>Stuttgart</td>
<td>0.978</td>
<td>0.191</td>
</tr>
<tr>
<td>Freiburg</td>
<td>0.984</td>
<td>0.134</td>
</tr>
<tr>
<td>Mannheim</td>
<td>0.974</td>
<td>0.153</td>
</tr>
<tr>
<td>General</td>
<td>0.975</td>
<td>0.180</td>
</tr>
</tbody>
</table>

Notes: RMSE, root mean squared error. 
*Mean value of validation results for all three stations.

Table 5. Diffuse fraction obtained from sunshine duration for the Heliosat-2 model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Calibration</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>$p_2$</td>
<td>$p_3$</td>
</tr>
<tr>
<td>Stuttgart</td>
<td>0.237</td>
<td>−1.020</td>
</tr>
<tr>
<td>Freiburg</td>
<td>0.197</td>
<td>−1.002</td>
</tr>
<tr>
<td>Mannheim</td>
<td>0.203</td>
<td>−0.986</td>
</tr>
<tr>
<td>General</td>
<td>0.229</td>
<td>−1.011</td>
</tr>
</tbody>
</table>

Notes: RMSE, root mean squared error. 
*Mean value of validation results for all three stations.

For all three stations the performances are considerably good. Although the parameters vary for different stations, they are not very sensitive. For the sake of simplification, the general parameter sets obtained by fusing the data from all three stations is used, which gives comparable results as the parameter set from the split-sample approach.

4.5.2. Mapping spatial global radiation

Once the global radiation and the diffuse fraction on horizontal surface are available, Equations (33) and (35) can be used to map spatial radiation, which can be regarded as a downscaling procedure, by which the radiation on the horizontal surface acts as the regional value. Figure 9 shows the comparison of the mapping results of actual global radiation in the year 2002 simulated by the two models. Figure 9a shows the $R^2$ and RMSE of the two methods throughout the year. The $R^2$ is considerably high, and most of time is over 0.80, except for a short period around summer time. Even in summer seasons, the $R^2$ never goes below 0.60 and the RMSE is also relatively small, which demonstrates the results of the two methods agree well with each other. A detailed inspection reveals that the error mainly originate from the deviation of the regional radiation (radiation on horizontal surface) calculated by the two methods, and the error from diffusion model is of minor importance. Figure 9b illustrates the relative variability of the spatial radiation in terms of NIQR. Again the two models agree well with each other. The spatial variability is much stronger in winter than in summer. The mapping results of actual radiation show that the simple AP approach performs reasonably well compared with the more elaborate Heliosat-2 model.
Figure 9. Comparison of spatial mapping results of Heliosat-2 and Angström–Prescott (AP) for Talhausen catchment. (a) $R^2$ is denoted as circle and root mean squared error (RMSE) is denoted as square. (b) Relative spatial variability is measured by normalized interquantile range (NIQR).

5. Conclusion
The article first reviews the solar radiation modelling steps and approaches which are summarized into a flowchart. The review is not meant to be thorough, but just a guideline for model selection for hydrological applications. Based on the review, the remote-sensing-based Heliosat-2 model and the adapted AP model are selected for mapping the actual spatial solar radiation. The models are tested in three steps: in the first step, the geometric modelling component and clear-sky parameterization of the Heliosat-2 model are tested against SA and observed data. It is shown that even though $rsun$ is conceptually simpler than SA, its results are more stable for arbitrarily oriented surfaces, whereas SA is error prone in days of equinox and solstice. In the second step, scale sensitivity of the $rsun$ model in terms of shading and spatial variability is tested, which reveals that shading is insignificant except at very fine scale and a critical threshold scale should be maintained in order to resolve the spatial variation of radiation. It is 100 m in the example case. It also demonstrates that the resampling procedure from finer DEM to obtain coarser DEM may cause erratic results at some points. In the last step, both the Heliosat-2 model and the AP model are validated for horizontal surface under cloudy conditions at three weather stations in Southern Germany, and both of them perform very well. The two models are also applied to inclined surface under overcast conditions. Although accounting for atmospheric effects, particularly clouds, involves a lot of simplifying approximations and requires measured data either in the form of satellite imagery or ground observations, the two selected models have reasonably characterized the spatial variability caused by topography. The AP model applying sunshine duration is conceptually very simple, but it has been shown that the sunshine duration is a good surrogate for the cloud parameterization. For ungauged basins without any ground observations, the Heliosat-2 method is a capable method for actual solar radiation mapping. The strong spatial variability of solar radiation in winter time, as demonstrated by the two models, may have essential effects on energy-limited hydrological processes, such as snowmelt and evapotranspiration, and should be considered explicitly in hydrological modelling at a proper scale.

Acknowledgements
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Nomenclature

- $\beta$: Slope [°]
- $B$: Beam irradiation [W h m$^{-2}$]
- $b$: Beam irradiance [W m$^{-2}$]
- $C_k$: Cloudness index [-]
- $D$: Diffuse irradiation [W h m$^{-2}$]
- $d$: Diffuse irradiance [W m$^{-2}$]
- $\delta_e$: Effective optical depth of all substances [-]
- $\delta_i$: Substance-specific optical thickness [-]
- $f_{af}$: Portion of forward scattered radiation [-]
- $f_{as}$: Scattered fraction by aerosol [-]
- $F_d$: Diffuse angular function [-]
- $F_M$: Muneer’s tilt factor [-]
- $G$: Global irradiation [W h m$^{-2}$]
- $H$: Extraterrestrial irradiation [W h m$^{-2}$]
- $I$: Solar irradiance [W m$^{-2}$]
- $I_{SC}$: Solar radiation constant, $\approx 1353$ W m$^{-2}$
- $K_b$: Direct radiation transmittance [-]
- $K_d$: Diffuse radiation fraction [-]
- $K_{hay}$: Hay’s sky clarity index [-]
- $K_t$: Clearness index [-]
- $K_w$: Clear-sky index [-]
- $K_{bw}$: Beam component of clear-sky index [-]
- $K_{dw}$: Diffuse component of clear-sky index [-]
- $m_i$: Optical air mass [h]
- $m_r$: Rayleigh optical air mass [h]
- $m_r(i)$: Relative optical air mass [h]
- $n_d$: Actual sunshine duration [h]
- $N_{dh}$: Potential sunshine duration [h]
- $\omega_{sr}$: Sunrise hour angle [°]
- $\omega_{ss}$: Sunset hour angle [°]
- $p$: Empirical coefficient [-]
- $q$: Empirical coefficient [-]
- $\tau_a$: Transmittance of aerosol [-]
- $\tau_e$: Effective transmittance of all substances [-]
- $\tau_{gas}$: Transmittance of uniformly mixed gas [-]
- $\tau_{oz}$: Transmittance of ozone [-]
- $\tau_r$: Transmittance of Rayleigh scattering [-]
- $\tau_{wa}$: Transmittance of water vapour [-]
- $\theta$: Zenith angle for arbitrary surface [°]
- $T_{Ln}$: Linke turbidity [-]
- $T_M$: Muneer’s tilt factor [-]
- $\psi_\beta$: Diffuse coefficient [-]
$\rho$ Albedo $[-]$

$\xi$ Radiation ratio on inclined/horizontal surface $[-]$

**Subscripts**

$\beta$ Inclined surface

c Clear-sky condition

$h$ Horizontal surface

$w$ Cloudy-sky condition

**References**


